Real Number System

The real number system is comprised of the set of real numbers and the arithmetic operations of addition and multiplication (subtraction, division and other operations are derived from these two). The rules and relationships that govern the real number system are the basis for most algebraic manipulations.

Properties of Real Numbers

All real numbers have the following properties:

(1) Reflexive Property

For any real number a, $a = a$.
Example: $3 = 3$, $y = y$, $x + z = x + z$ (x, y and z are real numbers)

(2) Symmetric Property

For any real numbers a and b, if $a = b$, then $b = a$.
Example: If $3 = 1 + 2$, then $1 + 2 = 3$

(3) Transitive Property

For any real numbers a, b and c, if $a = b$ and $b = c$, then $a = c$.
Example: If $2 + 3 = 5$ and $5 = 1 + 4$, then $2 + 3 = 1 + 4$.

(4) Substitution Property

For any real numbers a and b, if $a = b$, then a may be replaced by b, and b may be replaced by a, in any mathematical statement without changing the meaning of the statement.

Example: If $a = 3$ and $a + b = 5$, then $3 + b = 5$.

(5) Trichotomy Property

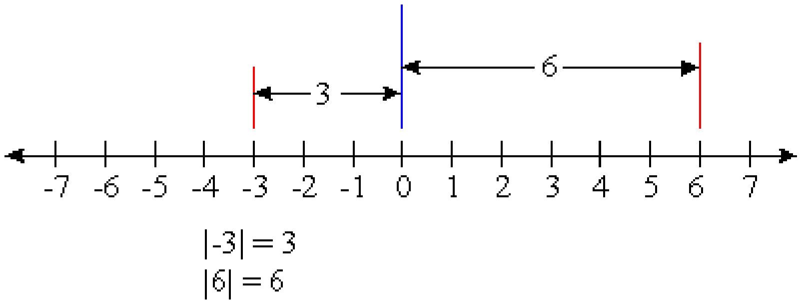
For any real numbers a and b, one and only one of the following conditions holds:

(1) a is greater than b ($a > b$)
(2) a is equal to b ($a = b$)
(3) a is less than b ($a < b$)

Example: $3 < 4$, $4 + 2 = 6$, $7 > 5$

Absolute Values

The absolute value of a real number is the distance between its corresponding point on the number line and the number 0. The absolute value of the real number a is denoted by $|a|$.



From the diagram, it is clear that the absolute value of nonnegative numbers is the number itself, while the absolute value of negative integers is the negative of the number. Thus, the absolute value of a real number can be defined as follows:

For all real numbers a,
(1) If a >= 0, then |a| = a.
(2) If a < 0, then |a| = -a.

Examples:
| 2 | = 2
| -4.5 | = 4.5
| 0 | = 0