Radical Equations

Radicals appear in a wide variety of applications, so we need to spend time learning to solve radical equations.

Solving Radical Equations

There are several steps to follow in solving radical equations

i. Rewrite the equation so that the radical is on one side of the equal sign and everything else is on the other side
ii. Square both sides of the equation (eliminates the radical)
iii. Solve remaining equation
iv. Check your solution

Examples:

$$\sqrt{x} = 3$$
$$\sqrt{x}^2 = 3^2$$
$$x = 9$$

check: $\sqrt{9} = 3$

$$1 + \sqrt{2t+5} = 4$$
$$\sqrt{2t+5} = 3$$
$$\left(\sqrt{2t+5}\right)^2 = 3^2$$
$$2t + 5 = 9$$
$$2t = 4$$
$$t = 2$$

check: $1 + \sqrt{2(2)+5} = 1 + \sqrt{4+5} = 1 + \sqrt{9} = 1 + 3 = 4$

Checking your solution in this type of problem is especially important. The act of squaring to remove the radical causes problems. Solutions to the equation that we created by squaring may not be solutions to the equation we had before squaring (extraneous solution)

Example:

$$\sqrt{w-1}+w=3$$
$$\sqrt{w-1}=3-w$$
$$\left(\sqrt{w-1}\right)^2=(3-w)^2$$
$$w-1=9-6w+w^2$$
$$w^2-7w+10=0$$
$$(w-2)(w-5)=0$$
$$(w-2)=0 \Rightarrow w=2$$
$$(w-5)=0 \Rightarrow w=5$$

Check $w = 2$

$$\sqrt{2-1}+2$$
$$\sqrt{1}+2$$
$$1+2$$
$$3$$

TRUE

Check $w = 5$

$$\sqrt{5-1}+5$$
$$\sqrt{4}+5$$
$$2+5$$
$$7$$

NOT TRUE