

## Properties of Radicals

We know  $\sqrt{36} = 6$  but we can also evaluate it this way

$\sqrt{36} = \sqrt{4 \cdot 9} = \sqrt{4} \cdot \sqrt{9} = 2 \cdot 3 = 6$ , from here we can see a basic property of roots.

### Roots of Products Property

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

This property is very useful in simplifying radical expressions.

### To simplify radical expressions

Write the expression under the radical as the product of two more terms that can be more easily dealt with and then apply the Roots of Products Property.

Examples:

$$\sqrt{75} = \sqrt{25 \cdot 3} = \sqrt{25} \cdot \sqrt{3} = 5\sqrt{3}$$

$$\sqrt{18} = \sqrt{9 \cdot 2} = \sqrt{9} \cdot \sqrt{2} = 3\sqrt{2}$$

$$\sqrt[3]{16} = \sqrt[3]{8 \cdot 2} = \sqrt[3]{8} \cdot \sqrt[3]{2} = 2\sqrt[3]{2}$$

If there are variables in the expression, I find it easier to separate the variables from the constants and deal with each part separately.

To deal with the roots of variable expression, it is easier to perform a simple 4<sup>th</sup> grade division with remainders.

$\sqrt[3]{t^{32}}$ , divide  $32 \div 3 = 10 \text{ R}2$  (10 becomes the power outside the radical and 2 becomes the power inside the radical), therefore we get  $\sqrt[3]{t^{32}} = t^{10} \cdot \sqrt[3]{t^2}$

Examples:

$$\sqrt{9x^5} = \sqrt{9} \sqrt{x^5} = 3x^1 \sqrt{x^2}$$

$$\sqrt[3]{16t^4u^5} = \sqrt[3]{16} \cdot \sqrt[3]{t^4} \cdot \sqrt[3]{u^5} = 2\sqrt[3]{2} \cdot t\sqrt[3]{t} \cdot u\sqrt[3]{u^2} = 2tu \cdot \sqrt[3]{2tu^2}$$

$\sqrt[3]{x^7y^9z^{16}}$ , break this up

$$\sqrt[3]{x^7} = x^2 \cdot \sqrt[3]{x}$$

$$\sqrt[3]{y^9} = y^3$$

$$\sqrt[3]{z^{16}} = z^5 \cdot \sqrt[3]{z}$$

and get  $x^2y^3z^5 \cdot \sqrt[3]{xz}$

### Roots of Quotients Property

$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ , you can handle the numerator and denominator separately

Examples:

$$\sqrt{\frac{25}{16}} = \frac{\sqrt{25}}{\sqrt{16}} = \frac{5}{4}$$

$$\sqrt{\frac{11}{49}} = \frac{\sqrt{11}}{\sqrt{49}} = \frac{\sqrt{11}}{7}$$

$$\sqrt{\frac{7}{x^2}} = \frac{\sqrt{7}}{\sqrt{x^2}} = \frac{\sqrt{7}}{x}$$

$$\sqrt[5]{\frac{19}{32c^{10}}} = \frac{\sqrt[5]{19}}{\sqrt[5]{32c^{10}}} = \frac{\sqrt[5]{19}}{2c^2}$$