Division of Radicals

Division of radicals follow the same kind of rules as multiplication

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}} \quad , b \neq 0$$

Examples:

$$\frac{\sqrt{27}}{\sqrt{3}} = \sqrt{\frac{27}{3}} = \sqrt{9} = 3$$

$$\frac{\sqrt{75x^3}}{\sqrt{3x}} = \sqrt{\frac{75x^3}{3x}} = \sqrt{25x^2} = 5x$$

$$\frac{\sqrt[3]{32u^7}}{\sqrt[3]{4uv^3}} = \sqrt[3]{\frac{32u^7}{4uv^3}} = \sqrt[3]{\frac{8u^6}{v^3}} = \frac{2u^2}{v}$$

On occasion, you will find a problem leaves you with a radical in the denominator, i.e. $\frac{2}{\sqrt{7}}$. It is generally easier to work with the fraction if there are no radicals in the denominator. We use a process called "rationalizing the denominator" to free the denominator of radicals.

Rationalizing the denominator

Rationalizing the denominator utilizes the following two rules (that we already know)

$$Rule\,1 \quad \sqrt{a} \cdot \sqrt{a} = \sqrt{a^2} = a$$

$$Rule\,2 \quad \left(\sqrt{a} + \sqrt{b}\right)\left(\sqrt{a} - \sqrt{b}\right) = a - b$$

Radical free

Examples:

1. $$\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{\sqrt{9}} = \frac{2\sqrt{3}}{3}$$

2. $$\frac{\sqrt{8}}{\sqrt{10}} = \sqrt{\frac{8}{10}} = \sqrt{\frac{4}{5}} = \frac{\sqrt{4}}{\sqrt{5}} = \frac{\sqrt{4}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

3. $$\frac{6}{\sqrt{3}+\sqrt{5}} = \frac{6}{\sqrt{3}+\sqrt{5}} \cdot \frac{\sqrt{3}-\sqrt{5}}{\sqrt{3}-\sqrt{5}} = \frac{6\left(\sqrt{3}-\sqrt{5}\right)}{3-5} = \frac{6\left(\sqrt{3}-\sqrt{5}\right)}{-2} = -3\left(\sqrt{3}-\sqrt{5}\right)$$

4. $$\frac{x}{2\sqrt{x}+1} = \frac{x}{2\sqrt{x}+1} \cdot \frac{2\sqrt{x}-1}{2\sqrt{x}-1} = \frac{x\left(2\sqrt{x}-1\right)}{2^2(\sqrt{x})^2-1^2} = \frac{x\left(2\sqrt{x}-1\right)}{4x-1}$$

Note: It is not wrong to leave a radical in the denominator (the math police will not come and arrest you, if you do), but most teachers prefer to see the answers in this form.