

## Complex Solutions to Quadratic Equations

If the discriminant of a quadratic equation ( $b^2 - 4ac$ ) is less than zero, then the quadratic equation  $ax^2 + bx + c = 0$  has solutions only in the complex number system.

Examples:

Solve  $x^2 + x + 1 = 0$ ,  $a = 1$ ,  $b = 1$ ,  $c = 1$

By the quadratic formula

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)} = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$$

$$x = \frac{-1}{2} + \frac{\sqrt{3}}{2}i \text{ or } x = \frac{-1}{2} - \frac{\sqrt{3}}{2}i$$

Solve  $2x^2 - 4x + 3 = 0$ ,  $a = 2$ ,  $b = -4$ ,  $c = 3$

By the quadratic formula

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(3)}}{2(2)} = \frac{4 \pm \sqrt{16-24}}{4} = \frac{4 \pm \sqrt{-8}}{4} = \frac{4 \pm 2i\sqrt{2}}{4} = \frac{2(2 \pm i\sqrt{2})}{4}$$

$$x = \frac{2 \pm i\sqrt{2}}{2}$$

$$x = 1 + \frac{\sqrt{2}}{2}i \text{ or } x = 1 - \frac{\sqrt{2}}{2}i$$