Division with Exponents

In order to evaluate $\frac{a^n}{a^m}$ we need to develop a few rules that will lead us to a some interesting properties of exponents.

Look at $\frac{x^5}{x^3} = \frac{x*x*x*x*x}{x*x*x} = \frac{x*x}{1} = x^2$ note:5 – 3 = 2

Therefore we have a rule $\frac{a^n}{a^m} = a^{n-m}$

Examples:

$\frac{5^6}{5^4} = 5^{6-4} = 5^2$

$\frac{6^4}{6^4} = 6^0$

also $\frac{6^4}{6^4} = 1$ therefore $6^0 = 1$

From here, we get $a^0 = 1$ **Zero Power Rule**

$\frac{5^4}{5^6} = 5^{4-6} = 5^{-2}$

also $\frac{5^4}{5^6} = \frac{5*5*5*5}{5*5*5*5*5*5} = \frac{1}{5*5} = \frac{1}{5^{-2}}$ therefore $5^{-2} = \frac{1}{5^2}$

leading us to the $a^{-n} = \frac{1}{a^n}$ **Negative Power Rule**

$(\frac{3}{4})^2 = (\frac{3}{4})(\frac{3}{4}) = \frac{3^2}{4^2} = \frac{9}{16}$

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giving us $(\frac{a}{b})^n = \frac{a^n}{b^n}$ **Power of a Quotient Rule**

Further Examples:

$$(\frac{x^4}{y^3})^5 = (\frac{x^{4*5}}{y^{3*5}}) = \frac{x^{20}}{y^{15}}$$

$$\frac{9x^2(y^4)^3}{(3x^2y^3)^2} = \frac{9x^2y^{12}}{9x^4y^6} = \frac{9}{9} * \frac{x^2}{x^4} * \frac{y^{12}}{y^6} = 1 * \frac{1}{x^2} * y^6 = \frac{y^6}{x^2}$$

note: the variable ends up on the side of the fraction line as the largest power

$$(x^{-3}y^2)^{-2} = x^{-3*-2}y^{2*-2} = x^6y^{-4} = \frac{x^6}{y^4}$$

note: the negative power causes the variable to change sides of the fraction line